# Week 8: Infinite baffle, electrical impedance, Theile-Small parameters

Microphone and Loudspeaker Design - Level 5

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# A weekly fact about Salford..!

*Did you know...*

- Broughton Suspension Bridge was an iron chain suspension bridge built in 1826 to span the River Irwell between Broughton and Pendleton, now in Salford, Greater Manchester, England. On 12 April 1831, the bridge collapsed, reportedly due to mechanical resonance induced by troops marching in step. As a result of the incident, the British Army issued an order that troops should "break step" when crossing a bridge.

# What are we covering today?

1. Infinite baffle loudspeaker
2. Q-Factor
3. Electrical impedance
4. Loudspeaker parameters

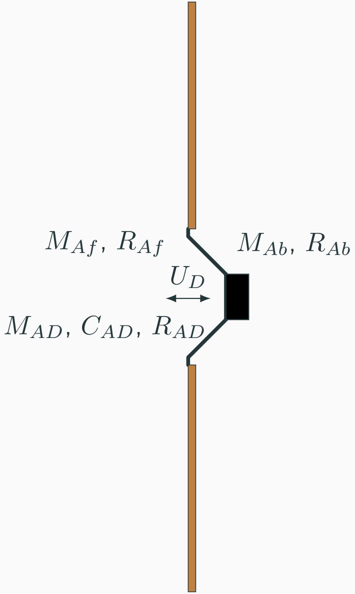
# Infinite baffle loudspeaker

# Infinite baffle: equivalent circuit acoustic loading

- We are ready to consider our first loudspeaker system - **an infinite baffle**
- When placed in an infinite baffle the diaphragm is loaded by the acoustic free space only
- We have derived this loading for a piston:

$$Z_{rad} \approx \frac{1}{2}\rho_0 c(ka)^2 + j\omega \frac{8}{3\pi}\rho_0 Sa \qquad (1)$$

- How might we include this in our equivalent circuit model?



# Infinite baffle: equivalent circuit acoustic loading

- Infinite baffle acoustic loading: $Z_{Af} = Z_{A,b} = R_A + j\omega M_A + \frac{1}{j\omega C_A}$

$$R_A = \frac{1}{2}\rho_0 c(ka)^2, \qquad M_A = \frac{8}{3\pi}\rho_0 S a, \qquad C_A = \infty \quad \left(C_A = \frac{V}{\rho_0 c^2}\right) \tag{2}$$

- Series resistor and inductor!

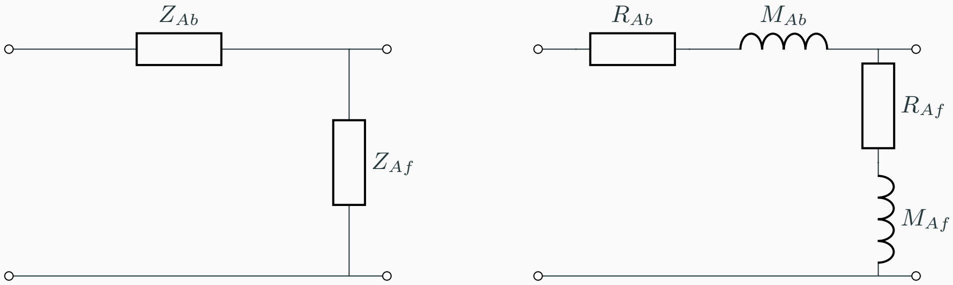



**Figure 2:** Equiv. circuit acoustic load for infinite baffle.

# Infinite baffle: complete equivalent circuit

- We now have an equivalent circuit for a loudspeaker in an infinite baffle.
- What happens if we consider very low frequencies?
  - Parallel capacitor impedance gets very large, circuit only sees the resistor.

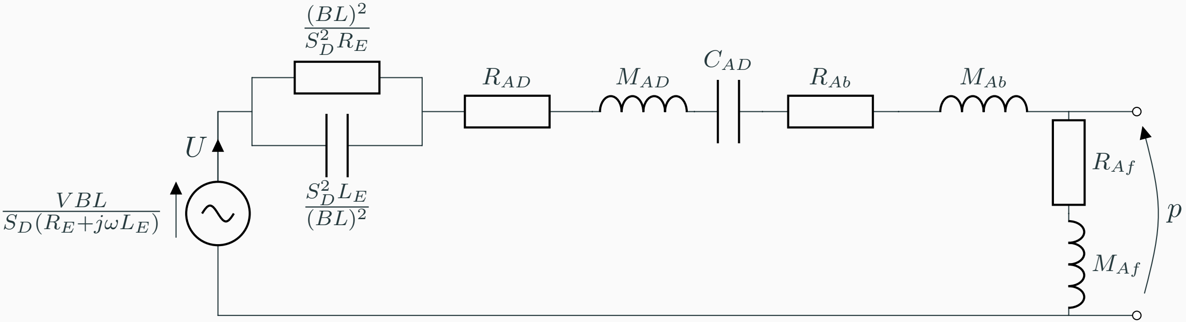



**Figure 3:** Equiv. impedance circuit for infinite baffle.

# Infinite baffle: complete equivalent circuit (very low freq.)

- Now we have a nice simple series circuit! Very easy to analyses.
- Lets group terms...

$$M_{AT} = M_{AD} + 2M_{Af} = M_{AS}, \quad C_{AT} = C_{AD}, \qquad R_{AT} = \frac{(BL)^2}{S^2 R_E} + R_{AD} + 2R_{Af} \quad (3)$$

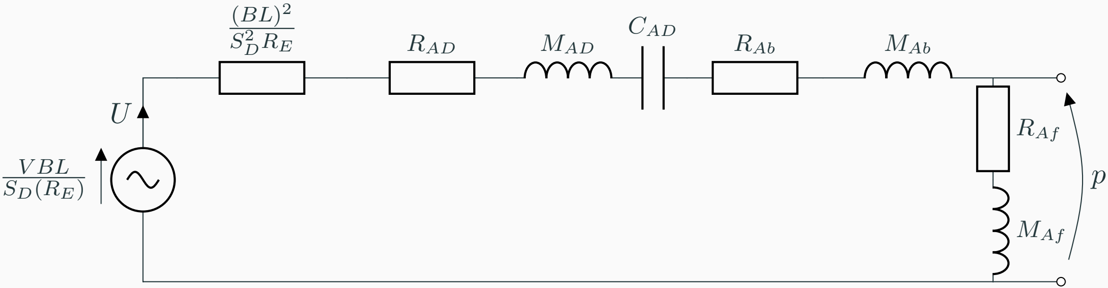



**Figure 4:** Equiv. impedance circuit for infinite baffle (very low freq. approx.).

# Infinite baffle: simple low frequency model

- Solve circuit for the volume velocity $U$ - use to drive piston radiation model.
- Recall impedance analogy: $V = IZ_T \rightarrow P = UZ_T$

$$V \sim P = \frac{VBL}{S_D R_E} \qquad Z_T = j\omega M_{AT} + \frac{1}{j\omega C_{AT}} + R_{AT} \tag{4}$$

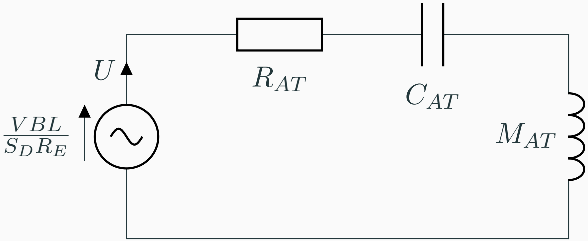



**Figure 5:** Equiv. impedance circuit for infinite baffle (very low freq. approx.).

# Infinite baffle: volume velocity

- Diaphragm volume velocity:

$$U = \frac{VBL}{S_D R_E} \frac{1}{\left(j\omega M_{AT} + \frac{1}{j\omega C_{AT}} + R_{AT}\right)} \tag{5}$$

- Factor out $j\omega M_{AT}$

$$U = \frac{VBL}{S_D R_E j\omega M_{AT}} \overbrace{\frac{1}{\left(1 + \frac{1}{(j\omega)^2 M_{AT} C_{AT}} + \frac{R_{AT}}{j\omega M_{AT}}\right)}}^{E(j\omega)} \tag{6}$$

- Re-parametrise $E(j\omega)$ in terms of:

$$\omega_s^2 = \frac{1}{M_{AT} C_{AT}} \qquad Q_{TS} = \frac{\omega_s M_{AT}}{R_{AT}} \tag{7}$$

# Infinite baffle: volume velocity

- Diaphragm volume velocity made up of two parts:

$$U = \underbrace{\frac{VBL}{S_D R_E j\omega M_{AT}}}_{\text{First order LPF}} E(j\omega) \quad (8)$$

$$E(j\omega) = \overbrace{\frac{1}{\left(1 + \frac{\omega_s}{j\omega}\frac{1}{Q_{TS}} - \frac{\omega_s^2}{\omega^2}\right)}}^{\text{Second order HPF}} \quad (9)$$

- What about the radiated pressure?

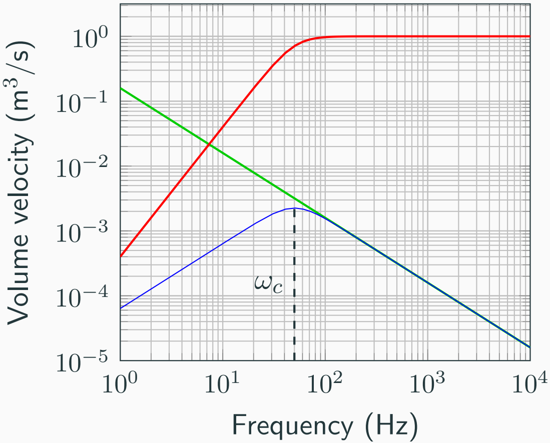



**Figure 6:** First and second order LPF/HPF terms in volume velocity

# Infinite baffle: radiated pressure

- Recall piston radiation:

$$p(r,t) = \frac{j\rho_0 ck}{4\pi r} U(\omega) \times \text{DF}(\,) \quad (10)$$

- What if we substitute in $U$:

$$U = \frac{VBL}{S_D R_E j\omega M_{AT}} E(j\omega) \quad (11)$$

- **CANCELLATION**

$$p(r,t) = \frac{\cancel{j\omega}\rho_0}{4\pi r} \frac{VBL}{S_D R_E \cancel{j\omega} M_{AT}} E(j\omega) \quad (12)$$

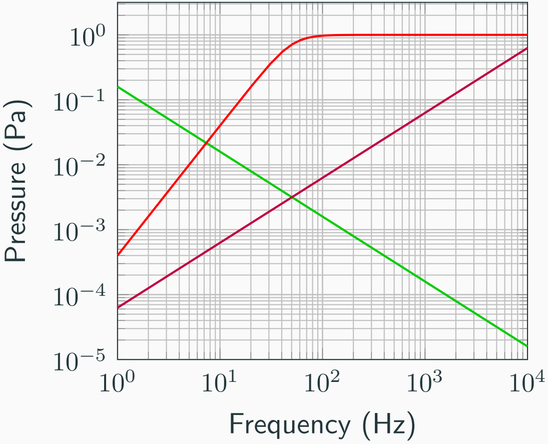



**Figure 7:** Radiated pressure terms

- Radiated pressure response

$$p(r,t) = \frac{\rho_0 V B L}{4\pi r S_D R_E M_{AT}} E(j\omega) \quad (13)$$

- Freq. dependence dictated by $E(j\omega)$:

$$E(j\omega) = \frac{1}{\left(1 + \frac{\omega_s}{j\omega}\frac{1}{Q_{TS}} - \frac{\omega_s^2}{\omega^2}\right)} \quad (14)$$

- Controlled by $\omega_s$ and $Q_{TS}$.

- Remaining terms describe sensitivity (i.e. pass-band level)

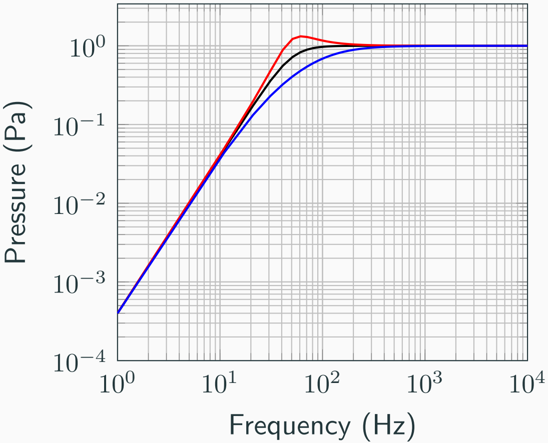



**Figure 8:** Example radiated pressure.

# Infinite baffle: sensitivity

- Radiated pressure response

$$p(r,t) = \frac{\rho_0 V BL}{4\pi r S_D R_E M_{AT}} E(j\omega) \xrightarrow{\omega >> \omega_c} \frac{\rho_0 V BL}{4\pi r S_D R_E M_{AT}} \tag{15}$$

- Definition of sensitivity,

$$\text{Sens.} = 20 \log_{10} \left( \frac{p}{p_0} \right)_{1m,1W} \tag{16}$$

- What voltage produces $1W$ of electrical power?

$$W = VI = V \frac{V}{R_E} = \frac{V^2}{R_E} = 1 \rightarrow V = \sqrt{R_E} \tag{17}$$

$$\text{Sens.} = 20 \log_{10} \left( \frac{\rho_0 \sqrt{R_E} BL}{p_0 4\pi r S_D R_E M_{AT}} \right) \Big|_{1m,1W} \tag{18}$$

# Q-Factor

# Q-factor: total

- We have the grouped terms:

$$M_{AT} = M_{AD} + 2M_{Af}, \qquad C_{AT} = C_{AD}, \qquad R_{AT} = \frac{(BL)^2}{S_D^2 R_E} + \overbrace{R_{AD} + \cancel{2R_{Af}}}^{R_{ADA}} \tag{19}$$

- Natural frequency of diaphragm

$$\omega_s = \sqrt{\frac{1}{M_{AT}C_{AT}}} \tag{20}$$

- Total Q-factor of diaphragm

$$Q_{TS} = \frac{1}{R_{AT}}\sqrt{\frac{M_{AT}}{C_{AT}}} = \left(\frac{(BL)^2}{S_D^2 R_E} + R_{AD}\right)^{-1}\sqrt{\frac{M_{AT}}{C_{AT}}} \tag{21}$$

## Q-factor: electrical and mechanical

- Useful to separate $Q_{TS}$ into two parts
  - Electrical Q-factor

$$Q_{ES} = \left( \frac{(BL)^2}{S_D^2 R_E} \right)^{-1} \sqrt{\frac{M_{AT}}{C_{AT}}} \tag{22}$$

  - Mechanical Q-factor

$$Q_{MS} = (R_{AD})^{-1} \sqrt{\frac{M_{AT}}{C_{AT}}} \tag{23}$$

- Total Q-factor given by combination

$$\frac{1}{Q_{TS}} = \frac{1}{Q_{ES}} + \frac{1}{Q_{MS}} \qquad \text{or} \qquad Q_{TS} = \frac{Q_{ES} Q_{MS}}{Q_{ES} + Q_{MS}} \tag{24}$$

# Electrical impedance

# Infinite baffle: equivalent circuit with transformers

- We would like to know what the impedance looks like from the electrical side.
  - Need to know this later when we design cross-over networks and compensation filters

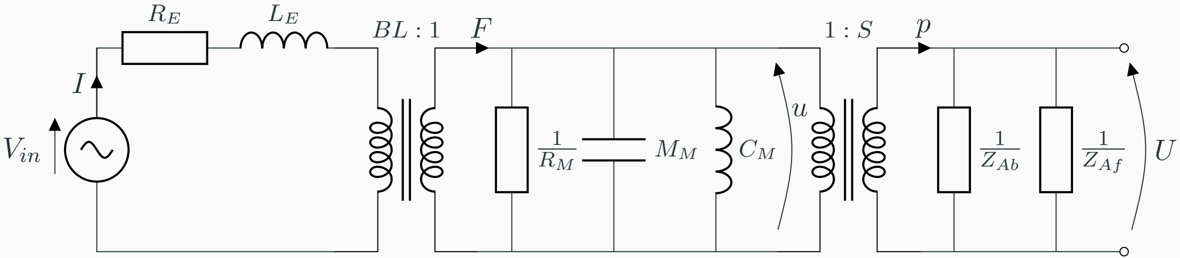



**Figure 9:** Equivalent circuit of a loudspeaker.

# Removing transformers: left to right

- Last time - moved everything into the acoustic domain
- Moving onto secondary winding side:
  - Scale impedances by reciprocal of turns ratio squared
  - Scale voltage source by reciprocal of turns ratio
  - Scale current sources by turns ratio

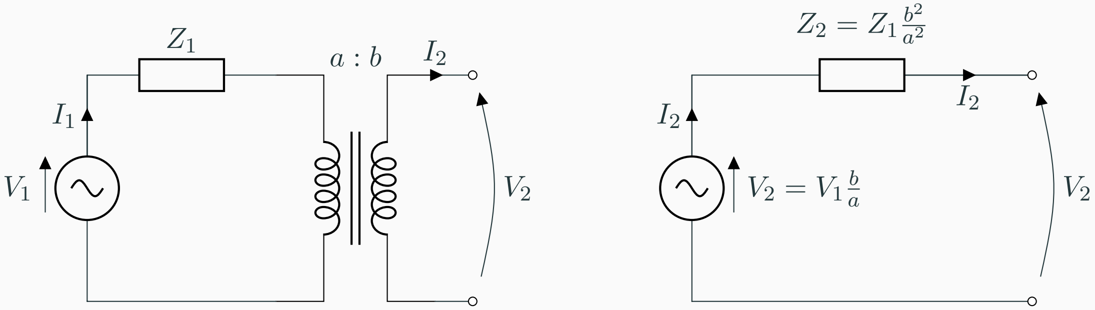



**Figure 10:** Moving electrical components across a transformer - left to right

# Removing transformers: right to left

- **This time - move everything to the electrical domain**
- Moving onto primary winding side:
  - Scale impedances by turns ratio squared
  - Scale voltage source by turns ratio
  - Scale current sources by reciprocal of turns ratio

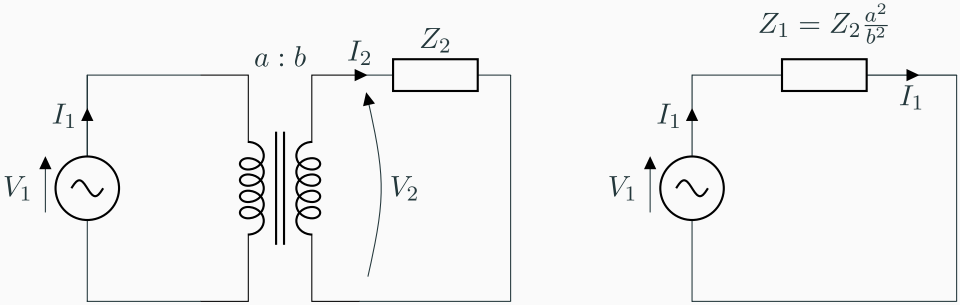



**Figure 11:** Moving electrical components across a transformer - left to right

# Infinite baffle: electrical domain

- We can use this circuit to determine the electrical impedance $Z_E$
- Split circuit into two parts: electrical and mechanical

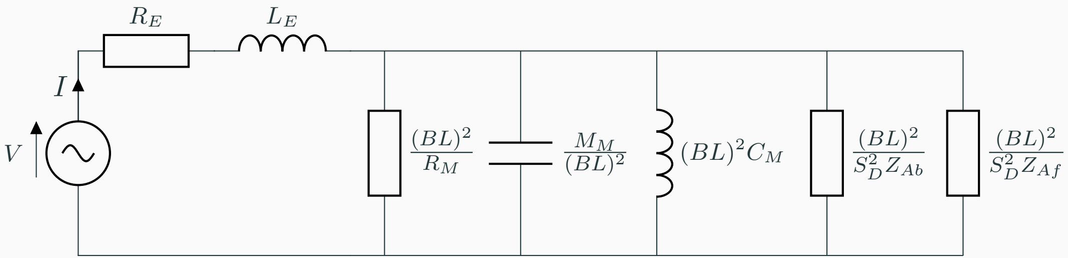



**Figure 12:** Equivalent circuit of a loudspeaker in the electrical domain.

- Split circuit into two parts: electrical and mechanical

$$Z_E = Z_{ES} + Z_{EM} \tag{25}$$

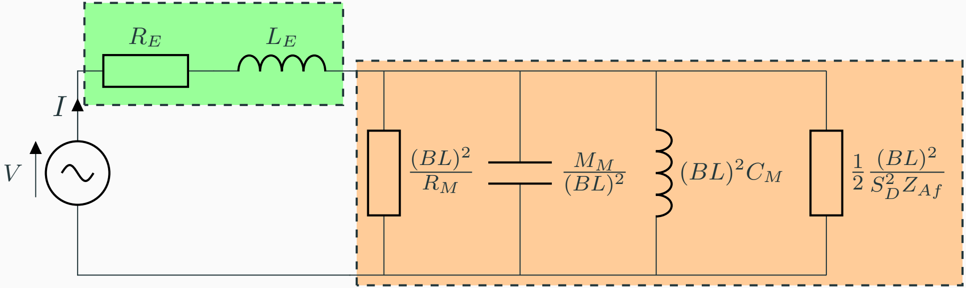



**Figure 13:** Equivalent circuit of a loudspeaker in the electrical domain.

# Infinite baffle: electrical domain

$$Z_{ES} = R_E + j\omega L_E \tag{26}$$

$$Z_{EM} = \left( \frac{1}{\frac{(BL)^2}{j\omega M_M}} + \frac{1}{\frac{(BL)^2}{R_M}} + \frac{1}{j\omega C_M (BL)^2} + \frac{1}{\frac{1}{2}\frac{(BL)^2}{S_D^2 j\omega M_{Af}}} + \frac{1}{\frac{1}{2}\frac{(BL)^2}{S_D^2 j\omega R_{Af}}} \right)^{-1} \tag{27}$$

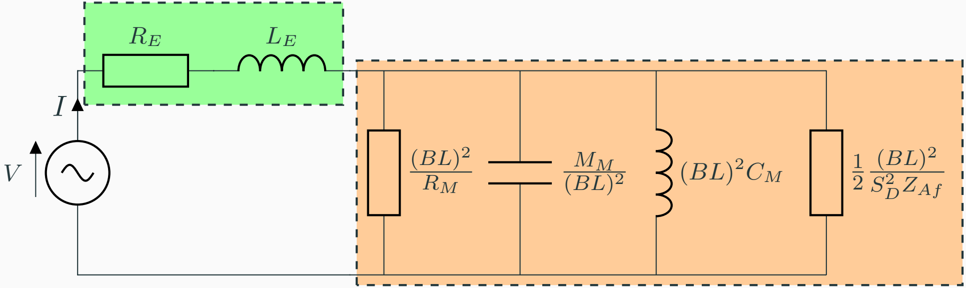



**Figure 14:** Equivalent circuit of a loudspeaker in the electrical domain.

- Combine: $M_{MS} = M_M + 2S_D^2 M_{Af}$, and $R_{MM} = R_M + 2S_D^2 R_{Af}$

$$Z_{EM} = \left( \frac{j\omega M_{MS}}{(BL)^2} + \frac{R_{MM}}{(BL)^2} + \frac{1}{j\omega C_M (BL)^2} \right)^{-1} \tag{28}$$

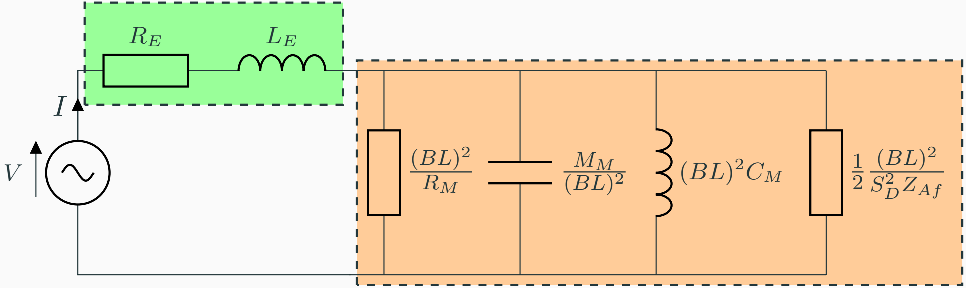



**Figure 15:** Equivalent circuit of a loudspeaker in the electrical domain.

# Infinite baffle: electrical domain

- Combine: $M_{MS} = M_M + 2S_D^2 M_{Af}$, and $R_{MM} = R_M + 2S_D^2 R_{Af}$

$$Z_{EM} = \left( \frac{j\omega M_{MS}}{(BL)^2} + \frac{R_{MM}}{(BL)^2} + \frac{1}{j\omega C_M (BL)^2} \right)^{-1} \tag{29}$$

- Re-parametrise in terms of:

$$\omega_c^2 = \frac{1}{M_{MS} C_M} \qquad Q_{MS} = \frac{\omega_s M_{MS}}{R_{MS}} \qquad R_{ES} = \frac{(BL)^2}{R_{MM}} \tag{30}$$

- Electrical impedance of infinite baffle loudspeaker:

$$Z_E = R_E + j\omega L_E + R_{ES} \left( \frac{\frac{1}{Q_{MS}} \frac{j\omega}{\omega_s}}{1 - \frac{\omega^2}{\omega_s^2} + \frac{1}{Q_{MS}} \frac{j\omega}{\omega_s}} \right) \tag{31}$$

- Electrical impedance of an infinite baffle loudspeaker:

$$Z_E = R_E + j\omega L_E + R_{ES}\left(\frac{\frac{1}{Q_{MS}}\frac{j\omega}{\omega_s}}{1 - \frac{\omega^2}{\omega_s^2} + \frac{1}{Q_{MS}}\frac{j\omega}{\omega_s}}\right) \tag{32}$$

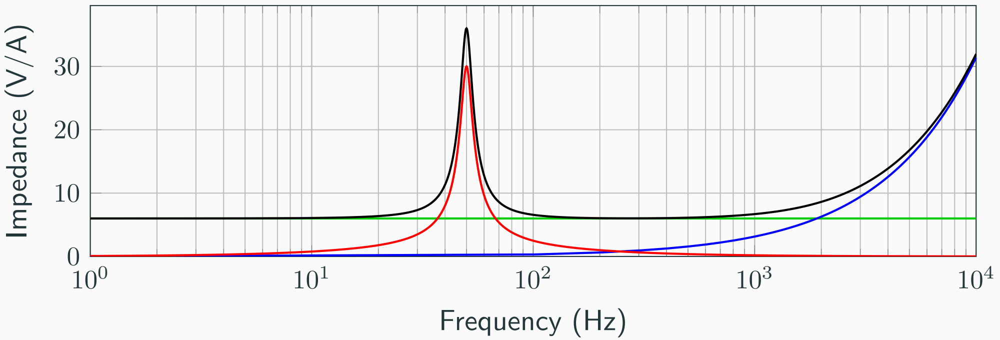



**Figure 16:** Electrical impedance of infinite baffle loudspeaker.

# Loudspeaker parameters

# Too many subscripts! Lets clarify.

- Lots of subscripts are popping up - gets a bit confusing.
- First subscript denotes the domain: *acoustic, mechanical, or electrical*
- Second subscript denotes the component: *diaphragm, speaker, air, etc.*
  - $M_{MS}$ - Mass of the speaker (diaphragm and air-load) in the mechanical domain
  - $M_{AD}$ - Mass of the diaphragm in the acoustic domain
  - $R_{MD}$ - Damping of the diaphragm suspension in the mechanical domain
  - $2R_{MA}$ - Damping due to air-loading in the mechanical domain
- Subscript $T$ denotes a 'total' quantity that includes all parts, for example:
  - $M_{AT} = M_{AD} + 2M_{AA}$ - Combined diaphragm/air load mass, in the acoustic domain.
  - $R_{MT} = R_{MD} + 2R_{MA} + R_{ME}$ - Combined damping due to suspension, air-loading, and electrical resistance, in the mechanical domain.
- For an infinite baffle subscripts $S$ and $T$ are somewhat interchangeable. This is not the case for sealed and vented cabinets.

# Loudspeaker parameters: fundamental

- Electrical impedance of an infinite baffle loudspeaker:

  $S_D$ Projected area of the driver diaphragm, in square meters

  $M_{MS}$ Mass of the diaphragm/coil, including acoustic load, in kilograms.

  $C_{MS}$ Compliance of the driver's suspension, in meters per newton

  $R_{MS}$ The mechanical resistance of a driver's suspension, including air load

  $L_E$ Voice coil inductance measured in millihenries

  $R_E$ DC resistance of the voice coil, measured in ohms

  $BL$ The product of magnet field strength in the voice coil gap and the length of wire in the magnetic field

- These parameters can be quite hard to measure directly...

# Loudspeaker parameters: Thiele-Small

- A complete low-frequency model of a loudspeaker driver unit can be described by just 6 parameters:

  $S_D$ Projected area of the driver diaphragm, in square meters

  $R_E$ DC resistance of the voice coil, measured in ohms

  $Q_{ES}$ Electrical Q-factor

  $Q_{MS}$ Mechanical Q-factor

  $f_s$ Diaphragm resonance frequency

  $V_{AS}$ Equivalent suspension volume - volume of air having the same acoustic compliance as the suspension

$$V_{AS} = C_{AS}\rho_0 c^2 = C_{MS} S_D^2 \rho_0 c^2 \quad (33)$$

- These are the **Thiele-Small parameters**. We can use these to determine the remaining parameters in our equivalent loudspeaker circuit!

# Loudspeaker parameters: Thiele-Small

$$C_{MS} = \frac{V_{AS}}{S_D^2 \rho_0 c^2} \quad \text{from} \quad V_{AS} = C_{AS}\rho_0 c^2 = C_{MS} S_D^2 \rho_0 c^2 \tag{34}$$

$$M_{MS} = \frac{1}{\omega_s^2 C_{MS}} \quad \text{from} \quad \omega_s = \sqrt{\frac{1}{M_{MS} C_{MS}}} \tag{35}$$

$$R_{MDA} = \frac{1}{Q_{MS}} \sqrt{\frac{M_{MS}}{C_{MS}}} \quad \text{from} \quad Q_{MS} = \frac{1}{R_{MDA}} \sqrt{\frac{M_{MS}}{C_{MS}}} \tag{36}$$

$$BL^2 = \frac{R_E}{\omega_s Q_{ES} C_{MS}} \quad \text{from} \quad Q_{ES} = \frac{R_E}{BL^2} \sqrt{\frac{M_{MS}}{C_{MS}}} \quad \text{and} \quad M_{MS} = \frac{1}{\omega_s^2 C_{MS}} \tag{37}$$

$$M_{MD} = M_{MS} - 2M_{MA} \quad \text{with} \quad M_{MA} = j\omega \frac{8\rho_0}{3\pi^2 a} \tag{38}$$